Workshop Calabi-Yau Motives

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Speakers:

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Paramodular forms as orthogonal modular forms A computational approach

The main goal of this talk is to describe a method to compute eigenvalues of (conjecturally) paramodular forms of weight 3.

In fact I will explain how to use neighbouring lattice methods to construct spaces of algebraic modular forms for orthogonal groups together with their Hecke operators.

In the case of SO(3) this construction is well known to correspond to classical modular forms of weight 2, by lifting to Brandt matrices (i.e. quaternionic modular forms) and the Eichler correspondence (aka Jacquet-Langlands). This involves the fact that Spin(3) is an inner form of SL(2).

In the case of SO(5) we have that Spin(5) is an inner form of Sp(2), whence it is fair to expect that quinary orthogonal modular forms correspond to Siegel modular forms of genus 2.
In this way, I learned from Hein and Voight, one can construct spaces of orthogonal modular forms that should correspond to spaces of paramodular forms of weight 3.

Note that these spaces will contain lifts of classical modular forms. However it was noted by Hein-Ladd-Tornaría that it is easy to compute a smaller subspace which will contain all the non-lifts in the space. I will also explain this idea.

Finally I will show the example of level N=61, which is the smallest conductor for which there is a non-lift paramodular form of weight 3.

In his phd thesis Jeffery Hein gives examples for prime levels p = 61, 73, 79, 89, 113, 167, 173, 197, and computes the Euler factors for primes up to 100. In the case of p = 61, I have also computed the eigenvalues for T_p for primes up to 1699.

This method is one quite promising way to construct a database of L-functions (on the automorphic side) corresponding to our objects of interest (a 'Gsp(2) Cremona table').